Vectors

Column Vectors

$\begin{pmatrix}3\\7\end{pmatrix} \rightarrow$ *3 units in the + ve x direction* & *7 units in the + ve y direction*

Unit vector is a vector with magnitude 1 unit. Hence, for $\underset{\sim}{\boldsymbol{a}}$,
the unit vector, $\hat{a} = \dfrac{\underset{\sim}{\boldsymbol{a}}}{|\underset{\sim}{\boldsymbol{a}}|}$

Two dimensional vectors in Cartesian form

In Cartesian form, $\underset{\sim}{\boldsymbol{i}}$ *represents the vector* $\begin{pmatrix}1\\0\end{pmatrix}$ & $\underset{\sim}{\boldsymbol{j}}$ *represents the vector* $\begin{pmatrix}0\\1\end{pmatrix}$.
Hence, *column vector* $\begin{pmatrix}5\\3\end{pmatrix}$ *can be written as* $5\ \underset{\sim}{\boldsymbol{i}} + 3\ \underset{\sim}{\boldsymbol{j}}$.

Example 1:

The coordinates of P, Q and R are (1, 0), (4, 2) and (5, 4) respectively. Use a vector method to determine the coordinates of S if
(a) PQRS is a parallelogram

Let the coordinates of S be (h, k)

Since PQRS is a parallelogram, $\overrightarrow{\mathbf{PQ}} = \overrightarrow{\mathbf{SR}}$.

$$\overrightarrow{\mathbf{OQ}} - \overrightarrow{\mathbf{OP}} = \overrightarrow{\mathbf{OR}} - \overrightarrow{\mathbf{OS}}$$

Example 2:

The vector $\overrightarrow{\mathbf{OA}}$ *has a magnitude* of 39 units and has the same direction as $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$. The vector $\overrightarrow{\mathbf{OB}}$ has a magnitude of 25 and has the same direction as $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$. Express $\overrightarrow{\mathbf{OA}}$ and $\overrightarrow{\mathbf{OB}}$ as column vectors and find the magnitude of $\overrightarrow{\mathbf{AB}}$.

Let $\underset{\sim}{\boldsymbol{c}}$ *be* $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$ & $\underset{\sim}{\boldsymbol{d}}$ *be* $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$.

$$\hat{\underset{\sim}{\boldsymbol{c}}} = \frac{\begin{pmatrix} 5 \\ 12 \end{pmatrix}}{\sqrt{5^2 + 12^2}} = \frac{\begin{pmatrix} 5 \\ 12 \end{pmatrix}}{13} = \frac{1}{13}\begin{pmatrix} 5 \\ 12 \end{pmatrix}$$

Since $\overrightarrow{\mathbf{OA}}$ *is in the same direction as* $\hat{\underset{\sim}{\boldsymbol{c}}}$, $\overrightarrow{\mathbf{OA}} = 39 \times \frac{1}{13}\begin{pmatrix} 5 \\ 12 \end{pmatrix} = 3\begin{pmatrix} 5 \\ 12 \end{pmatrix} = \begin{pmatrix} 15 \\ 36 \end{pmatrix}$

$$\hat{\underset{\sim}{\boldsymbol{d}}} = \frac{\begin{pmatrix} -3 \\ 4 \end{pmatrix}}{\sqrt{\left(-3^2 + 4^2\right)}} = \frac{1}{5}\begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

Since $\overrightarrow{\mathbf{OB}}$ *is in the same direction* as $\hat{\underset{\sim}{\boldsymbol{d}}}$, $\overrightarrow{\mathbf{OB}}$ =

Question 1:

The vector $\overrightarrow{\mathbf{OA}}$ has magnitude 100 and has the same direction as $\begin{pmatrix} 7 \\ 24 \end{pmatrix}$. Express $\overrightarrow{\mathbf{OA}}$ as a column vector. The vector $\overrightarrow{\mathbf{OB}}$ *is* $\begin{pmatrix} 24 \\ 99 \end{pmatrix}$. Obtain the unit vector in the direction of $\overrightarrow{\mathbf{AB}}$.

Question 2:

The position vectors of A, B and C relative to an origin O are $-\underset{\sim}{\boldsymbol{i}} + p\,\underset{\sim}{\boldsymbol{j}}$, $5\,\underset{\sim}{\boldsymbol{i}} + 9\,\underset{\sim}{\boldsymbol{j}}$ & $6\,\underset{\sim}{\boldsymbol{i}} + 8\,\underset{\sim}{\boldsymbol{j}}$ respectively. Determine the value of p for which A, B and C are collinear.

Give that D is a point of OC such that $\overrightarrow{\mathbf{OD}}$ *is* a unit vector, find the position vector of D relative to O.

Question 3:

The position vectors of two points A and B relative to an origin O are $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ & $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$ respectively. Find the position vector of the point S is

(a) S is the midpoint of AB,

(b) $\overrightarrow{\mathbf{SA}} = 3\overrightarrow{\mathbf{SB}}$,

(c) OSAB is a parallelogram.